Vidyamandir Classes
Gurukul for IITJEE & Medical Preparation

# SOLUTIONS

## Joint Entrance Exam | IITJEE-2019

### 10th APRIL 2019 | Morning Session

## Joint Entrance Exam | JEE Mains 2019

| PART-A | PHYSICS |
| --- | --- |

**1.(2)** In fiber optics, we use wavelength in range 1.3 – 1.6 $\mu$m (Infrared)

Radar stands for Radio aid to detection and ranging so, it uses radio waves.
Sonar uses high energy (high frequencies) sound waves for under-water research.
Mobile phones uses microwaves of wavelength of order of few metals.

**2.(4)** $r = \frac{mv}{qB}$ $\quad\therefore\quad$ $r \propto \frac{\sqrt{m}}{q}$ [K.E. is some for all particles]

$M_{H_e^{2+}} = 4m_p$

$q_{H_e^{+4}} = 2q_p$

$m_p > m_e$

$r_p = k\frac{\sqrt{m_p}}{q_p}$ ; $k = \sqrt{\frac{2(K.E)}{B}}$

$r_e = \frac{k\sqrt{m_e}}{q_e}$

$r_{Ae^{+4}} = \frac{k\sqrt{4\times m_p}}{2q_p} = k\sqrt{\frac{m_p}{q_p}}$ $\quad\therefore\quad$ $r_e < r_p = r_{He}$

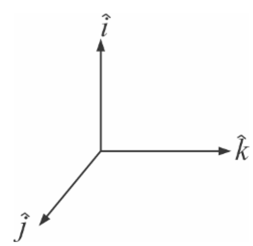
**3.(3)** Higher frequency heard will be when observer would move towards the source and lower frequency heard will be when observes moves away from the source.

$\therefore\quad 530 = 500\left[\frac{300 - V_{01}}{300 - 0}\right] \quad\Rightarrow\quad v_{01} = 18\ m/s$

Also, $480 = 500\left[\frac{300 - v_{02}}{300 - 0}\right] \quad\Rightarrow\quad v_{02} = 12\ m/s$

**4.(3)** $\vec{E} = E\hat{i}\cos(kz)\cos(wt)$

It is made by superposisson of 2 waves.




$\vec{E}_1 = \frac{E_0}{2}\hat{i}\cos(\mathrm{kz - wt})$

$\vec{E}_2 = \frac{E_0}{2}\hat{i}\cos(\mathrm{kz + wt})$

Corresponding $\vec{B}$

$\vec{B}_1 = \frac{E_0}{2C}\hat{j}\cos(\mathrm{kz - wt})$

$\vec{B}_2 = \frac{-E_0}{2C}\hat{j}\cos(\mathrm{kz + wt})$

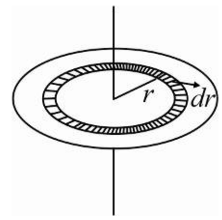
So, $\vec{B} = \vec{B}_1 + \vec{B}_2 = \hat{j}\frac{E_0}{2C}\times 2\cdot\sin(\mathrm{kz})\sin(\mathrm{wt})$

**5.(2)** Disc can be understood as the combination of co-axial rings.

M.I. of element ring of radius $r$ and infinitesimally small thickness $dr$ about the axis is

$dI = (dm).r^2 = \{(kr^2)2\pi r dr\}r^2$

$\therefore$ M.I. of disc $I = \int dI = \int_{r=0}^{R} kr^2.(2\pi r dr).r^2$




$\Rightarrow$ $I = 2\pi k R^6 / 6$ .... (1)

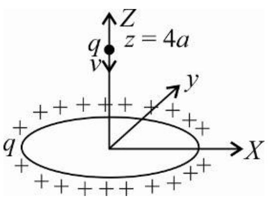
Also, mass of ring, $M = \int dm = \int_{r=0}^{R} (kr^2)(2\pi r dr)$

Or, $M = 2\pi k \frac{R^4}{4}$ ... (2)

From (1) & (2)

$I = \frac{4}{6}MR^2 = \frac{2}{3}MR^2$

**6.(2)**




From energy conservation,

$U_i + k_i = U_f + k_f$

$$\frac{k(q)(q)}{\sqrt{(3a)^2 + (4a)^2}} + \frac{1}{2}mv_{\min.}^2 = \frac{k(q)(q)}{3a} + 0 \quad \Rightarrow \quad V_{\min.} = \sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi \epsilon_0 a}\right)^{1/2}$$

**7.(2)** For series combination

$\frac{C_1C_2}{C_1 + C_2} = C_{eq} = \frac{80}{10} = 8$ ... (1)

For parallel combination

$C_1 + C_2 = C_{eq} = \frac{500}{10} = 50$ ... (2)

Solving (1) & (2)

$C_1 = 40\mu F$

$C_2 = 10\mu F$

**8.(3)** $Q = nC_V \Delta T$ (volume is onts.)

$n = \frac{67.2}{22.4} = 3$ (as molar volume of a gas at stp = 22.4 Lit.)

$C_V = \frac{fR}{2} = \frac{3R}{2}$

$\Delta T = 20°$

$Q = 3 \times \frac{3R}{2} \times 20 = 90R = 747.9\ J$

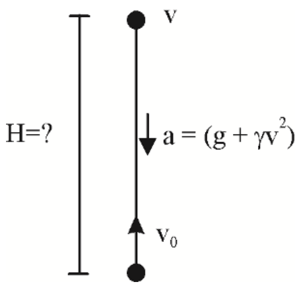
**9.(1)** $N_A = N_0 e^{-10\lambda t}$

$N_B = N_0 e^{-\lambda t}$

To find $t$ when $\dfrac{N_A}{N_B} = \dfrac{1}{e}$

$\Rightarrow \quad \dfrac{e^{-10\lambda t}}{e^{-\lambda t}} = \dfrac{1}{e} \Rightarrow e^{-9\lambda t} = e^{-1} \quad \Rightarrow \quad 9\lambda t = 1 \Rightarrow t = \dfrac{1}{9\lambda}$

**10.(2)**




$a = \dfrac{dv}{dt} = -(g + \gamma . v^2) \quad \Rightarrow \quad \int_{v_0}^{0} \dfrac{dv}{(g + \gamma v^2)} = \int_0^t dt \quad \Rightarrow \quad -\dfrac{1}{\gamma} \int_{v_0}^{0} \dfrac{dv}{v^2 + \left(\sqrt{g/r}\right)^2} = \int_0^t dt$

$\Rightarrow \quad t = \dfrac{1}{\gamma} \dfrac{1}{\sqrt{g/r}} \left[ \tan^{-1} \left[ \dfrac{v}{\sqrt{g/r}} \right] \right]_0^{v_0}$ Or $\quad t = \sqrt{\dfrac{1}{g\gamma}} \tan^{-1} \left[ \sqrt{\dfrac{\gamma}{g}} v_0 \right]$

**11.(3)** $\ln R$ varies linearly with $\dfrac{1}{T^2}$

So, $\ln R = -M\left(\dfrac{1}{T^2}\right) + c \qquad [m > 0, c > 0]$

Using $y = -mx + c$

$R = e^{c - \frac{m}{T^2}} = e^c . e^{-\frac{m}{T^2}}$

Or $R = R_0 e^{-m/T^2}$ type of form will do good

**12.(1)** From conservation of momentum

Along x-direction:

$M(10)\cos 30^\circ + (2M)(5)\cos 45^\circ = (2M).v_1 \cos 30^\circ + M.v_2 \cos 45^\circ$

$\Rightarrow \quad \sqrt{3} v_1 + \dfrac{v_2}{\sqrt{2}} = 5\sqrt{3} + 5\sqrt{2} \qquad \ldots (1)$

Along y-direction:

$(2M) \times (5)\sin 45^\circ - M(10)\sin 30^\circ = (2M)v_1 \sin 30^\circ - Mv_2 \sin 45^\circ \quad \Rightarrow \quad v_1 - \dfrac{v_2}{\sqrt{2}} = 5\sqrt{2} - 5 \qquad \ldots (2)$

Solving (1) & (2)

$v_1 = 6.5\ m/s$

$v_2 = 6.3\ m/s$

**13.(1)** J = Current density $= \frac{E}{\rho}$ (E = electric field, $\rho$ = resistive)

$\Rightarrow \quad E = \rho.\frac{I}{A}$ (I = current)

$\because \quad$ drift velocity $= v_d = \mu E$

$\Rightarrow \quad \mu = mobility = \frac{v_d}{E} = \frac{v_d A}{\rho I} = \frac{1.1 \times 10^{-3} \times \pi \times 25 \times 10^{-6}}{1.7 \times 10^{-8} \times 5} \cong 10.15 \times 10^{-1} = 1.015\ m^2/vs$

**14.(1)** $N_P =$ turns in primary = 300

$N_S$ turns in Secondary = 150 $\quad \because \quad \frac{E_p}{E_s} = \frac{N_p}{N_s} = \frac{300}{150} = 2$

Output power = $2.2\ kw$ $\quad \because \quad$ Output power $= E_s \cdot I_s = 2.2 \times 10^3$

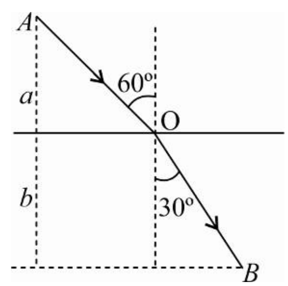
$\Rightarrow \quad E_s = \frac{2200}{I_s} = \frac{2200}{10} = 220v \quad \Rightarrow \quad E_p = 440\ v$

For Lossless transform
Input power = Output power

$\Rightarrow \quad E_p I_p = 2200 \quad \Rightarrow \quad I_p = \frac{2200}{440} = 5A$

**15.(3)** Optical path $= \mu \times$ Geometrical path

So, optical path will be $= 1 \times OA + \mu_{glass} \times (OB)$




From shells Law

$\sin 60° = \mu \sin 30° \Rightarrow \mu = \sqrt{3}$

So, optical path $= 2a + \sqrt{3} \times \frac{2b}{\sqrt{3}} = 2a + 2b$

**16.()** $K.E_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{1237}{260} - \frac{1237}{380} = 1.5\ eV$

**17.(4)** $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R}$

$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{-(\mu_2 - 1)}{R} \quad \therefore \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{(\mu_1 - 1) - (\mu_2 - 1)}{R} = \frac{\mu_1 - \mu_2}{R}$

$\therefore \quad F = \frac{R}{(\mu_1 - \mu_2)}$

**18.(2)** Modulating index $= m = \frac{\text{Amplitude of massage signal}}{\text{Amplitude of Carrier signal}} = \frac{1}{4} = 0.25$

Three frequency are obtained, $f_{carrier}, f_{carrier} + f_{massage}$

So, Board with = $2 f_{massage} = 200\ MHz$

**19.(4)** According to equation,

$e^{-0.1t} = \frac{1}{2}$ (drop to half of its initial value)

$\Rightarrow \quad 0.1 \times t = \ln 2 = 0.693 \quad \Rightarrow \quad t = 6.93 \text{ sec} \approx 7 \text{ sec}$

**20.(3)** $\vec{r} = x\hat{i} + y\hat{j}$

$\vec{\tau} = \vec{r} \times \vec{F}$ (At $t = 0$)

So, $a_x = \frac{d^2x}{dt^2} = -\omega_1^2 a \cos \omega_1 t$

$a_y = \frac{d^2y}{dt^2} = -\omega_2^2 b \sin \omega_2 t$

$\vec{F}_{\text{at } t=0} = m\vec{a} = -m\omega_1^2 a\ \hat{i}$

$\vec{r}_{\text{at } t=0} = (x_0 + a)\hat{i} + y_0\hat{j}$

So, $\vec{\tau} = \vec{r} \times \vec{F} = m\omega_1^2 a y_0 \hat{k}$

**21.(1)** $h = \frac{2T\cos\theta}{\rho g r}$

So, $h = \frac{2T_{Hg} |\cos(135^o)|}{\rho_{Hg}.g.r_1} = \frac{2T_{water}.\cos 0^o}{\rho_{water}.g.r_2}$

$\Rightarrow \quad \frac{r_1}{r_2} = \left(\frac{T_{Hg}}{T_{water}}\right)\frac{|\cos 135^o|}{\cos 0^o} \times \frac{\rho_{water}}{\rho_{Hg}} = 7.5 \times \frac{1}{\sqrt{2}} \times \frac{1}{13.6} \cong 0.4$

**22.(3)** $10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 60 \Rightarrow \frac{P_{out}}{P_{in}} = 10^6.$

$P_{\text{out}} = \Delta V_{\text{out}} \cdot \Delta I_C$

$P_{in} = \Delta V_{in} \cdot \Delta I_B$

$\because \quad V_{\text{out}} = I_C \cdot R_{\text{out}} \Rightarrow \Delta V_{\text{out}} = R_{\text{out}} \cdot \Delta I_C \qquad \because \quad V_{\text{in}} = I_B \cdot R_{\text{in}} \Rightarrow \Delta V_{\text{in}} = R_{\text{in}} \cdot \Delta I_B$

$\frac{R_{\text{out}}}{R_{\text{in}}} \cdot B^2 = 10^6$

$\beta = 100$

**23.()** For voltmeter $i_{g/\max}(R_g + R) = V_{\max.}$ or, $R_g = \frac{V_{\max.}}{i_{g,\max}} - R = \left(\frac{5}{10^{-4}} - 2\times 10^6\right) =$ negative

As an Ammeter

$i_G \times R_G = (i - i_a) \times S$

$S = \frac{i_G \times R_G}{i - i_G} = \frac{10^{-4} \times R_G}{(10^{-2} - 10^{-4})} = 10^{-2} R_G$ = negative

$\therefore$ No any option is possible. Some data is question error

**24.(3)** Collision frequency $= \pi d^2 \sqrt{2}\ V_{avg} \dfrac{nN_A}{V}$

$V_{avg}$ : Avg. speed

V : Volume of container

Putting values

Collision frequency $= 3.14 \times \left(0.3 \times 10^{-9}\right)^2 \times \sqrt{2} \times \sqrt{\dfrac{8}{3\pi}} \times V_{rms} \times \dfrac{1 \times 6.023 \times 10^{23}}{25 \times 10^{-3}}$ $\left\{ v(avg.) = \sqrt{\dfrac{8}{3\pi}} V_{rms} \right\}$

$$= 3.14 \times \left(0.3 \times 10^{-9}\right)^2 \times \sqrt{2} \times \sqrt{\frac{8}{3\pi}} \times 200 \times \frac{1 \times 6.023 \times 10^{23}}{25 \times 10^{-3}} \approx 1.8 \times 10^9 \approx 10^{10} \text{ order}$$

**25.(1)** $g' = \dfrac{g}{\left(1 + \dfrac{h}{\text{Re}}\right)^2}$

$$4.9 = \frac{9.8}{\left(1 + \frac{h}{\text{Re}}\right)^2} \qquad \Rightarrow \qquad \left(1 + \frac{h}{\text{Re}}\right) = \sqrt{2} = 1.414$$

$$\Rightarrow \qquad \frac{h}{\text{Re}} = 0.414 \qquad \Rightarrow \qquad h = 0.414 \times \text{Re} = 0.414 \times 6.4 \times 10^6 = 2.6 \times 10^6$$

**26.(3)** $\dfrac{W}{Q} = \dfrac{nR\Delta T}{{}^{n}C_P \cdot \Delta T} = \dfrac{nR\Delta T}{n(C_1 + R)\Delta T} = \dfrac{R}{C_v + R}$ $C_v$ : molar specific heat of gas

$= \dfrac{R}{\left(\dfrac{C_v}{n} + R\right)} = \dfrac{nR}{C_v + nR}$ ; $C_v$: specific heat of 'n' moles of gas

**27.(1)** From conservation of Angular momentum about the axis;

$Li = L_f$

$$\Rightarrow \qquad I_1\omega_1 + \frac{I_1}{2} \times \frac{\omega_1}{2} = \left(I_1 + \frac{I_1}{2}\right)\omega_f \qquad \Rightarrow \qquad \omega_f = \frac{\frac{5}{4} I_1\omega_1}{\frac{3}{2} I_1} = \frac{5}{6}\omega_1$$

$$\therefore \qquad E_f - E_i = \frac{1}{2}\left(I_1 + \frac{I_1}{2}\right)\omega_f^2 - \left[\frac{1}{2} I_1\omega_1^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right)^2\right] = \frac{1}{2} \times \frac{3I_1}{2}\left(\frac{5}{6}\omega_1\right)^2 - \frac{9}{16} I_1\omega_1^2 = -\frac{I_1\omega_1^2}{24}$$

**28.(2)** Current through battery

$i = \dfrac{1.5 + 1.5}{\left(\dfrac{15 \times 10}{15 + 10} + 2 + 2r\right)}$ $\therefore$ Current through $10\Omega$ resistor;

$i_{10\Omega} = \dfrac{15}{15 + 10} \times i = \dfrac{3}{5} \times i$ $\therefore$ voltage across $10\Omega$ resistor $= \left(\dfrac{3}{5} i\right) \times 10 = 2 \text{ (Given)}$

$$\Rightarrow \qquad \frac{3}{5} \times \frac{3 \times 10}{(6 + 2 + 2r)} = 2 \qquad \Rightarrow \qquad (8 + 2r) = \frac{90}{10} = 9 \qquad \Rightarrow \qquad r = \frac{1}{2} = 0.5\Omega$$

**29.(2)** Observation should match the condition of balanced wheatstone bridge arrangement.

$\therefore \quad \frac{R}{l} = \frac{X}{(100-l)} \quad \Rightarrow \quad X = \frac{R}{l} \times (100-l)$

From Reading 1:

$X = \frac{1000}{60} \times (100-60) = 666.67\Omega$

From Reading 2:

$x = \frac{100}{13} \times (100-13) = 669.23\Omega$

From Reading 3:

$X = \frac{10}{1.5} \times (100-1.5) = 656.67\Omega$

From Reading 4:

$X = \frac{1}{1} \times (100-1) = 99\Omega$

So, reading (4) gives the most in consistent result.

**30.(2)** Force on wire C is zero when net magnetic field due to wires A and B is zero.

This is possible only when wire C is either left of A or right of B.

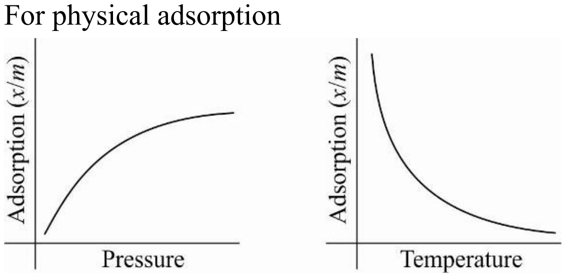
In these cases;

$B_1 = B_2 \quad \Rightarrow \quad \frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(d+x)}$ or $\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{(x-d)} \quad \Rightarrow \quad x = \frac{I_1 d}{I_2 - I_1}$ or $x = \frac{I_1 d}{(I_1 - I_2)}$

## Joint Entrance Exam | JEE Mains 2019

| PART-B | CHEMISTRY |
|---|---|

**1.** For physical adsorption



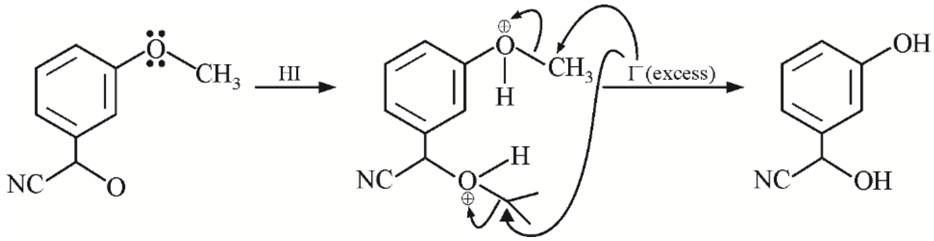

**2.** Fact (Refer NCERT)

**3.** Refer NCERT (Metallurgy)

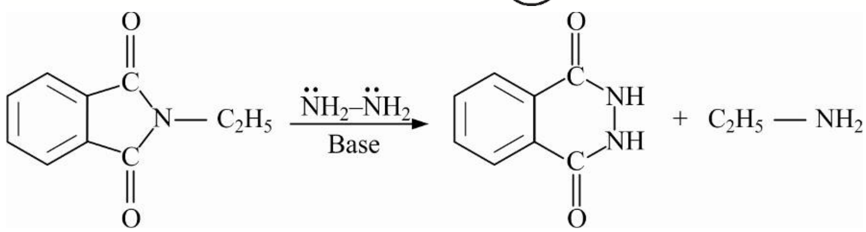
**4.(B)** Amylopectin is branched polymer of α-D-glucose having $C_1 - C_4$ and $C_2 - C_6$ linkage.

**5.** Theory

**6.**




**7.**




**8.** **(A)**

$$\underset{\substack{40\text{ m mol}\\(\text{excess})}}{H_2SO_4} + \underset{\substack{40\text{ m mol}\\(\text{Limiting reagent})}}{2NaOH} \rightarrow Na_2SO_4 + 2H_2O$$

After reaction

20 mmol    0    20 mmol    40 mmol

Total volume $= 400 + 400 = 800$ mmol

Number of moles of $H_2SO_4 = 20\,\text{m mol}$

Number of moles of $H^+$ion $= 2 \times 20$ m mol $= 40$ m mol

$$[H^+] = \frac{40}{800} = 0.05\text{ M}$$

$$pH = -\log[H^+] = -\log(0.05) = 2 - \log 5 = 2 - 0.7 = 1.3$$

$$pH = 1.3$$

**(B)** $K_\omega = [H^+][OH^-]$

As T increases, $K_\omega$ also increases

**(C)** $pH = 5$ and $K_a = 10^{-5}$

$\Rightarrow \quad -\log[H^+] = 5$

$[H^+] = 10^{-5}$

Weak monobasic acid 'HA' having concentration 'C'

| | HA(aq) | $H^+(aq)$ | $+ A^{-1}(aq)$ |
|---|---|---|---|
| At $t = 0$ | C | 0 | 0 |
| At eqlibrium | $C - C\alpha$ | $C\alpha$ | $C\alpha$ |

$$K_a = \frac{[H^+][A^{-1}]}{[HA]} = \frac{C\alpha}{C(1-\alpha)}$$

$\alpha \rightarrow$ degree of dissociation

$\therefore \quad K_a = \dfrac{C\alpha^2}{1-\alpha} = 10^{-5}$ … (1) (given)

$[H^+] = C\alpha = 10^{-5}$ … (2)

From equation (1) & (2)

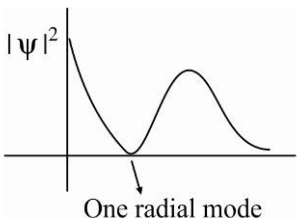
$\dfrac{\alpha}{1-\alpha} = 1 \quad \Rightarrow \quad \alpha = 1-\alpha$

$\alpha = \dfrac{1}{2}$

$\%\ \alpha = \dfrac{1}{2} \times 100 = 50\%$

**8.** **(D)** It is applicable.

**9.**



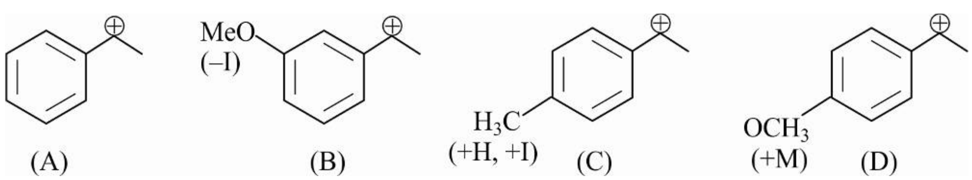

And radial mode

Number of radial node $= n - 1 - 1$

$n \rightarrow$ principal quantum number

$l \rightarrow$ azimuthal quantum number

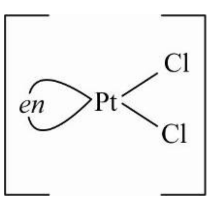
**10.**



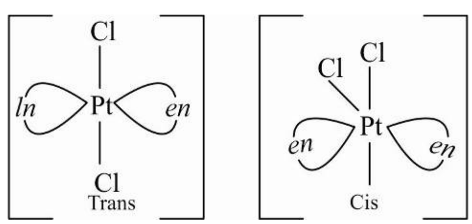

Rate of $S_N1 \propto$ stability of carbocation $\propto \dfrac{+M, +H, +I}{-M, -I}$

B < A < C < D

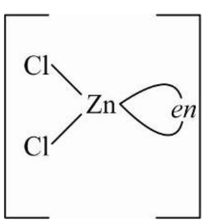
**11.** (a)



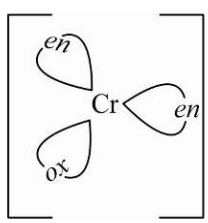

(b)



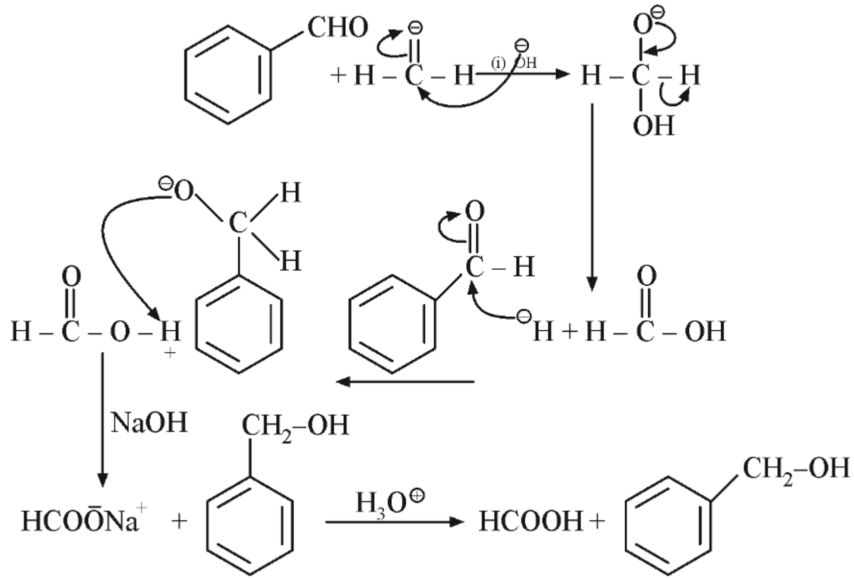

**(c)**



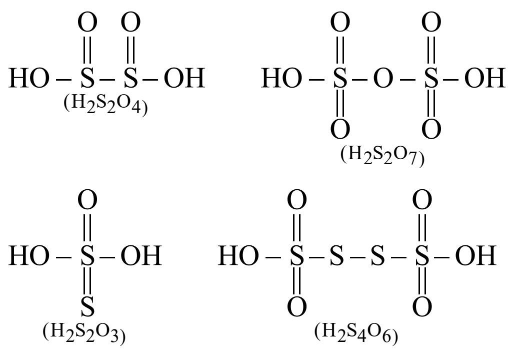

(d)



**12.**




**13.**




**14.(D)** Nylon is condensation polymer while all other are addition polymers.

**15.** Fact

**16.**

$$C_xH_y + \left(x + \frac{y}{4}\right)O_{2(g)} \longrightarrow xCO_{2(g)} + \frac{y}{2}H_2O_{(l)}$$

| | $C_xH_y$ | $O_2$ | $CO_2$ | $H_2O$ |
|---|---|---|---|---|
| Initial condition | 10ml | 55ml | 0 | 0 |
| After reaction | 0 | 0 | 40 ml | – |

From stoichiometry concept,

$$10\left(x + \frac{y}{4}\right) = 55 \qquad \Big| \qquad 10x = 40 \Rightarrow x = 4$$

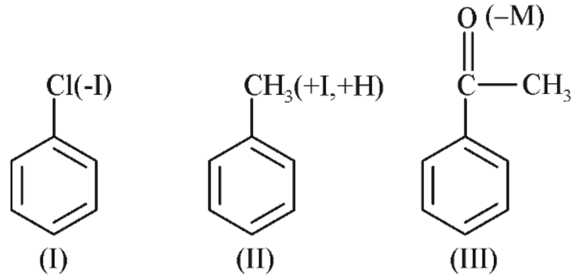
$$\Rightarrow \quad 10x + 10\frac{y}{4} = 55$$

$$10x + \frac{10y}{4} = 55$$

$$y = \frac{15 \times 4}{10} \quad \therefore \quad y = 6$$

Hence $C_xH_y \Rightarrow C_4H_6$

**17.** Rate of electrophilic Aromatic substitution reaction $\propto$ electron density on benzene ring




Hence $-Cl$ & $-\overset{\overset{\large O}{||}}{C}-CH_3$ groups are electron withdrawing and $-CH_3$ is electron releasing group

III < I < II

**18.** $P_0 \rightarrow$ Vapour pressure of pure solvent

$P_s \rightarrow$ Vapour pressure of pure solution.

Number of moles of urea $= \dfrac{0.60}{60} = 0.01\,\text{mol}$

Number of moles of water $= \dfrac{360}{18} = 20\,\text{mol}$

$P_o - P_S \Rightarrow$ Lowering in Vapour- pressure

We know

$$\frac{P_o - P_S}{P_0} = X_{solute}$$

$$P_o - P_s = \left(\frac{n_{solute}}{n_{solute} + n_{solvent}}\right) \times P_o = \frac{0.01}{0.01 + 20} \times 35 = \frac{0.01}{20.01} \times 35 = 0.0174$$

**19.** Fact

**20.** Up to one hour

$N = N_0 e^t$

At $t = 1\,\text{hour}$

$N = N_0 e$

After one hour

$$\frac{dN}{dt} = -5N^2$$

$$\int_{N_o e}^{N} \frac{dN}{N^2} = -5\int_1^t dt$$

$$\left[\frac{1}{N}\right]_{N_o e}^{N} = 5[t]_1^t$$

$$\frac{1}{N} - \frac{1}{N_o e} = 5(t-1)$$

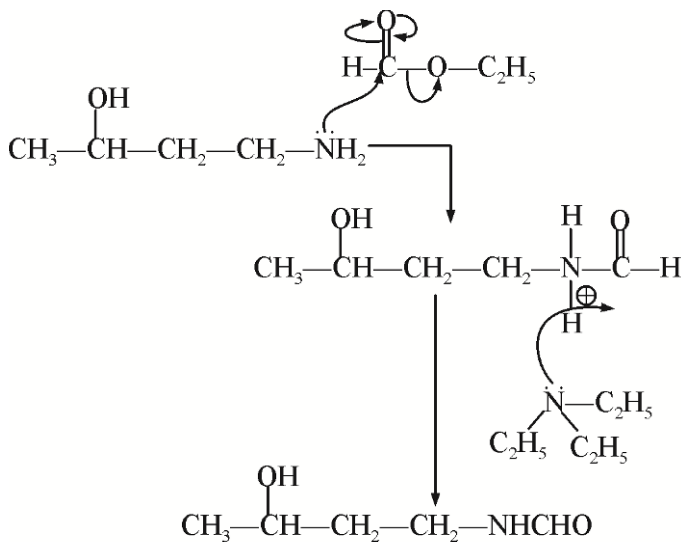
$$\frac{1}{N} = 5t - 5 + \frac{1}{N_0 e}$$

Multiply $N_o$ both side

$$\frac{N_0}{N} = 5N_0 t + \left(\frac{1}{e} - 5N_o\right)$$

$y = mx + c$

**21.**



**22.** For spontaneous process

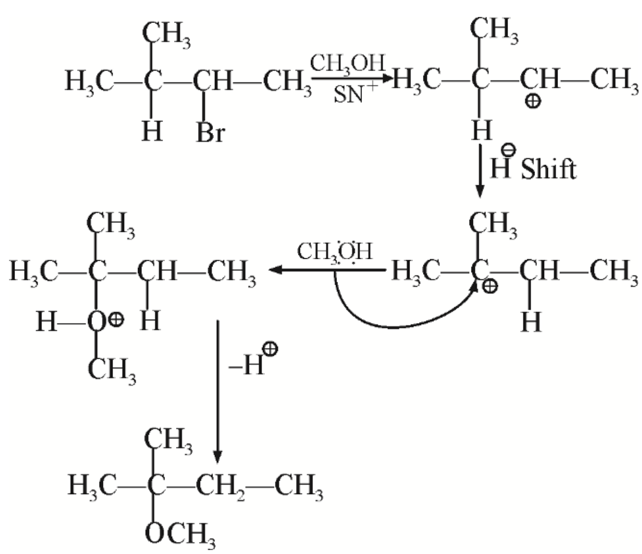
$\Delta G = (\Delta H - T\Delta S)$ should be negative

$\therefore \quad \Delta G < 0$

**23.** Conductivity $\propto$ Concentration
(K)

Molar conductivity $\propto \dfrac{1}{\text{Concentration}}$

**24.**



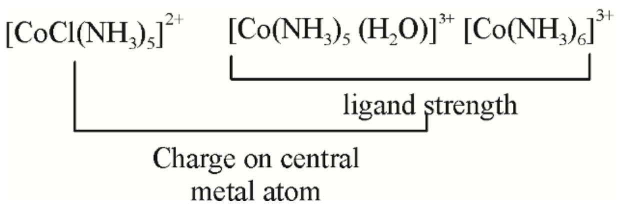
**25.** $N^{3-}, O^{2-}, F^{-}$ and $Na^{+}$ $\Rightarrow$ 10 electron

**26.** Absorb light energy $\propto \dfrac{1}{\text{wavelength of light}}$

Absorb energy $\propto$ splitting energy $(\Delta_o)$

$\propto$ charge on central metal atom

$\propto$ ligand strength




**27.** `a' represent attractive force

`b' represent size of molecule

Any molecule having


**28.** By MOT

$O_2$

$\sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2pz^2\ \pi 2px^2 = \pi 2py^2\ \pi^* 2px^1 = \pi^* 2py^1$

**29.** $T_i^{+2} \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^2$

no. of unpaired $e^- = 2$

$T_i^{+3} \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^1$

no. of unpaired $e^- = 1$

$V^{+2} \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^3$

no. of unpaired $e^- = 3$

$Sc^{+3} \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$

no. of unpaired $e^- = 0$

spin-only magnetic moment $\propto$ no. of unpaired $e^-$

**30.** Fact

## Joint Entrance Exam | JEE Mains 2019

| PART-C | MATHEMATICS |
|---|---|

**1.(3)** $\lim\limits_{n\to 0} f(0-h) = 1+(p+1) = p+2$

$\lim\limits_{n\to 0} f(0+h) = \frac{1}{2}$

$f(0) = q$

$p+2 = q$ and $q = \frac{1}{2}$

$\left(-\frac{3}{2}, \frac{1}{2}\right)$

**2.(2)** Here $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

Expanding the determinant, we get

$\Delta_1 = -x^3$ (which is independent of $\theta$)

Similarly, $\Delta_2$, $\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix} = -x^3$

So, $\Delta_1 + \Delta_2 = -2x^3$

**3.(2)** $\dfrac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})(\alpha-\beta)^{24}} \Rightarrow \dfrac{\alpha^{12}\beta^{12}}{(\alpha-\beta)^{24}} \Rightarrow \dfrac{(\alpha\beta)^{12}}{[(\alpha-\beta)^2]^{12}} \Rightarrow \dfrac{(-2\sin\theta)^{12}}{\left((\sin\theta)^2+8\sin\theta\right)^{12}}$

$\Rightarrow \dfrac{2^{12}\sin^{12}\theta}{\sin^{12}\theta(\sin\theta+\theta)^{12}} \Rightarrow \dfrac{2^{12}}{(\sin\theta+\theta)^{12}}$

**4.(2)** $|z| = \dfrac{2}{\sqrt{a^2+1}} = \sqrt{\dfrac{2}{5}}$

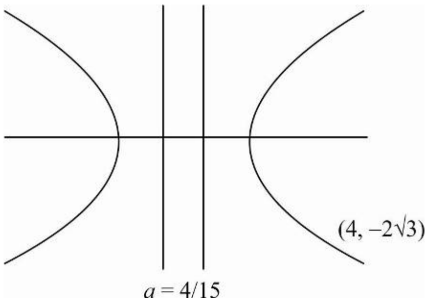
$\Rightarrow \dfrac{4}{a^2+1} = \dfrac{2}{5} \Rightarrow a^2+1 = 10 \Rightarrow a^2 = 9 \Rightarrow a = 3$ (since $a > 0$)

$z = \dfrac{2i}{3-i}$

$\bar{z} = \dfrac{-2i}{3+i} \Rightarrow \bar{z} = \dfrac{-2i(3-i)}{10}$

$\bar{z} = \dfrac{-6i-2}{10} \Rightarrow \bar{z} = -\dfrac{1}{5} - \dfrac{3}{5}i$

**5.(4)**




$$\frac{16}{a^2}-\frac{12}{b^2}=1$$

$$\frac{16}{a^2}-\frac{12}{a^2(e^2-1)}=1$$

$$16(e^2-1)-12=a^2(e^2-1) \qquad \text{since, } \frac{a}{e}=\frac{4}{\sqrt{5}}$$

$$16e^2-16-12=\frac{16}{5}e^2(e^2-1)$$

$$80e^2-80-60=16(e^4-e^2)$$

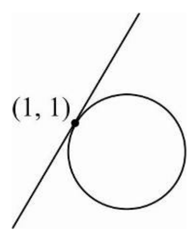
$$80e^2-140=16e^4-16e^2$$

$$16e^4-96e^2+140=0$$

$$4e^2-24e^2+35=0$$

**6.(3)** Family of circles touching the line $y-x=0$ at $(1,1)$

$(x-1)^2+(y-1)^2+\lambda(y-x)=0$ ......... (i)




Since circle passes through $(1,-3)$

So, it must satisfy equation (i)

$0^2+16+\lambda(-3-1)=0$

$\lambda=4$

On putting the value of $\lambda$ in equation (i)

$(x-1)^2+(y-1)^2+4(y-x)=0$

$x^2-y^2-6x+2y+2=0$

Hence radius of the circle $=\sqrt{9+1-2}=\sqrt{8}=2\sqrt{2}$

**7.(3)** Equation of common chord is given by $S_1-S_2=0$

Here equation of common chord is given by

$4kx+\frac{1}{2}y+k+\frac{1}{2}=0$ ... (i)

Now this line (i) must be identical to

$4x+5y-k=0$ ... (ii)

Dividing (i) by $k$

$4x+\frac{1}{2k}y+1+\frac{1}{2k}=0$ ........ (iii)

On comparing (i) and (iii)

$\frac{1}{2k}=5$ and $-k=1+\frac{1}{2k}$

$k=\frac{1}{10}$ and $-2k^2=2k+1$

$k=\frac{1}{10}$ and $2k^2+2k+1=0$ has imaginary roots

No value of *k*.

**8.(3)** For the given system of equations

$$D=\begin{vmatrix}1&1&1\\1&2&2\\1&3&\lambda\end{vmatrix}=(\lambda-3),\quad D_1=\begin{vmatrix}5&1&1\\6&2&2\\\mu&3&\lambda\end{vmatrix}=4(\lambda-3)$$

$$D_2=\begin{vmatrix}1&5&1\\1&6&2\\1&\mu&\lambda\end{vmatrix}=\lambda-\mu+4$$

$$D_3=\begin{vmatrix}1&1&5\\1&2&6\\1&3&\lambda\end{vmatrix}=\mu-7$$

For infinite many solution $D=D_1=D_2=D_3=0$

So, $\lambda=3$ and $\mu=7$ so $\lambda+\mu=10$

**9.(4)** $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+......+\frac{(2n)^{1/3}}{n^{4/3}}\right)$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{(n+r)^{1/3}}{n^{4/3}}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^{1/3}\left(1+\frac{r}{n}\right)^{1/3}}{n^{4/3}}$$

$$\int_0^1(1+x)^{1/3}dx$$

$$\left[\frac{(1+x)^{4/3}}{4/3}\right]_0^1 \Rightarrow \frac{2^{4/3}}{4/3}-\frac{1}{4/3} \Rightarrow \frac{3}{4}\cdot2^{4/3}-\frac{3}{4}$$

**10.(1)** $\frac{dy}{dx}=(\tan x-y)\sec^2 x \Rightarrow \frac{dy}{dx}+y\sec^2 x=\tan x\sec^2 x$

It is a L.D.E

I.F $=e^{\int\sec^2 x\,dx}=e^{\tan x}$

$y(e^{\tan x}) = \int e^{\tan x} \cdot \tan x \cdot \sec^2 x\, dx$

Put $\tan x = t$

$y(e^{\tan x}) = \int e^t \cdot t\, dt$

$y(e^{\tan x}) = e^t \cdot t - \int e^t dt + c$

$y(e^{\tan x}) = \tan x \cdot e^{\tan x} - e^{\tan x} + c$

$y(x) = \tan x - 1 + ce^{-\tan x}$

Now $y(0) = 0$

$0 = 0 - 1 + c$

$c = 1$

$y(x) = \tan x - 1 + e^{-\tan x}$

Now $y\left(-\frac{\pi}{4}\right)$

$\Rightarrow \quad y\left(-\frac{\pi}{4}\right) = -1 - 1 + e^{+1} \quad \Rightarrow \quad y\left(-\frac{\pi}{4}\right) = e - 2$

**11.(3)** $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to 1} \frac{x^3 - k^3}{x^2 - k^2} \quad \Rightarrow \quad 4(1)^3 = \lim_{x \to k} \frac{(x-k)(x^2 + k^2 + kx)}{(x-k)(x+k)}$

$\Rightarrow \quad 4 = \lim_{x \to k} \frac{(x^2 + k^2 + kx)}{(x+k)} \quad \Rightarrow \quad 4 = \frac{3}{2}k \quad \Rightarrow \quad k = \frac{8}{3}$

**12.(1)** Given equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ... (i)

Equation of tangent at $\left(3, -\frac{9}{2}\right)$

$\frac{3x}{a^2} + \frac{\left(-\frac{9}{2}\right)y}{b^2} = 1$

But the tangent is given by

$x - 2y = 12$ ... (ii)

So (i) and (ii) must be identical

$\frac{a^2}{3} = 12$ and $-\frac{2}{9}b^2 = -6$

$a^2 = 36$ and $b^2 = 27$

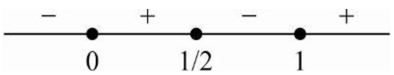
and length of $L.R = \frac{2b^2}{a} = \frac{2 \times 27}{6} = 9$

**13.(3)** Here $f(x)=e^x - x$ and $g(x)=x^2-x$

Let $h(x)=fog(x)=e^{x^2-x}-(x^2-x)$

$h'(x)=e^{x^2-n}(2x-1)-(2x-1)$

$h'(x)=(2x-1)(e^{x^2-x}-1)$



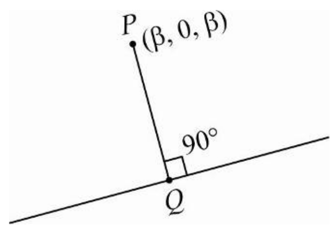

So, $n \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$

**14.(1)** Given six digits are 0, 1, 2, 5, 7 and 9
If (|(sum of odd places digit) - (even places digit)|) is divisible by 11, then the given number is divisible by 11.
Let one such type of number is *a b c d e f*
Then $|(a+c+e)-(b+d+f)|$ must be divisible by 11.
It is possible if $a, c, e \in \{2, 1, 9\}$ and $b, d, f \in \{0, 5, 7\}$
Now we can arrange the given digit in $3! \times 2! \times 2! - 2! \times 3! = 60$

**15.(3)** Given line is $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ and the point is $(\beta, 0, \beta)$ where $\beta \neq 0$.

Let coordinate of $Q(\lambda, 1, -\lambda-1)$

$(\lambda-\beta)\cdot 1+(1-0)\cdot 0+(-\lambda-1-\beta)(-1)=0$




From here $\lambda = -\frac{1}{2}$

So, the point $Q$ becomes

$\left(-\frac{1}{2}, 1, -\frac{1}{2}\right)$

Now $\sqrt{\left(\beta+\frac{1}{2}\right)^2+1+\left(\beta+\frac{1}{2}\right)^2}=\sqrt{\frac{3}{2}}$

So, $\beta=-1$

**16.(4)** By truth table option 4 is a tautology.

**17.(1)** $\int_0^{2\pi}[\sin 2x(1+\cos 3x)]dx$

Let $I=\int_0^{2\pi}[\sin 2x(1+\cos 3x)]dx$ ... (1)

$I=\int_0^{2\pi}[\sin 2(2\pi-x)(1+\cos 3(2\pi-x))]dx$

$I=\int_0^{2\pi}[-\sin 2x(1+\cos 3x)]dx$ ... (ii)

(1) + (2)

$2I=\int_0^{2\pi}(-1)\,dx$ $\quad [x]+[-x]=-1$

$2z=-2\pi \quad \Rightarrow \quad I=-\pi$

**18.(3)** $T_r = \dfrac{(2r+1)(1^3+2^3+......+r^3)}{1^2+2^2+......+r^2}$

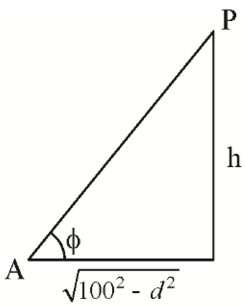
$$T_r = \frac{(2r+1)\left(\frac{r(r+1)}{2}\right)^2}{\frac{r(r+1)(2r+1)}{6}}$$

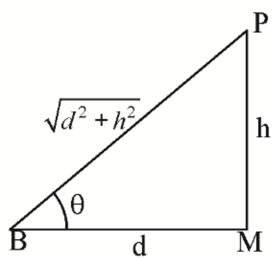
$$T_r = \frac{3}{2}r(r+1)$$

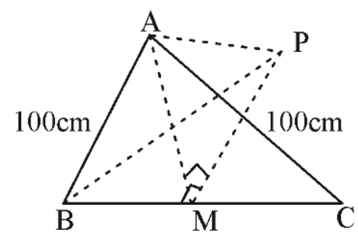
Sum up to 10th term $= \dfrac{3}{2}\displaystyle\sum_{r=1}^{10} r(r+1) = 660$

**19.(3)** Let h(m) be the height of the tower

Say BM = d




$\theta = \operatorname{cosec}^{-1} 2\sqrt{2}$

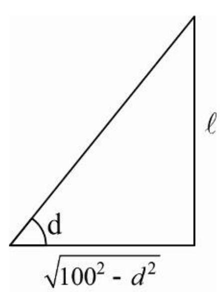
$$2\sqrt{2} = \frac{\sqrt{d^2+h^2}}{h}$$

$7h^2 = d^2$ … (1)

$$cot\,\phi = \frac{\sqrt{(100)^2 - d^2}}{h}$$

$$3\sqrt{2} = \frac{\sqrt{(100)^2 - d^2}}{h}$$

$18h^2 = 100^2 - d^2$ … (2)




Put the value of $d^2$ from (1) and (2)

$18h^2 = 100^2 - 7h^2 \Rightarrow 25h^2 = 100 \times 100$

$h = 20\,m$

**20.(4)** Given expression is $(1+ax+bx^2)(1-3x)^{15}$

Coff of $x^2=0$

Coff of $x^3=0$

Coff of $x^2={}^{15}C_2\times 9+b-{}^{15}C_1\times 3a$

$945+b-45a=0$ (given) … (1)

Coff of $x^3={}^{15}C_3(-27)-{}^{15}C_1\times 3b+{}^{15}C_2\times 9a$

$\Rightarrow$ 945a – 456 = 12285 (given) … (2)

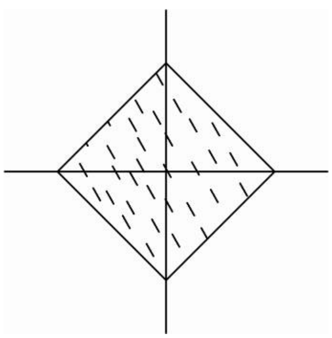
On solving (1) and (2) we get

a = 28, b = 315

**21.(2)** Here it is given that $|x-y|\le 2$ … (1)

and $|x+4|\le 2$ … (2)

Combinating 2



Square of side length $2\sqrt{2}$

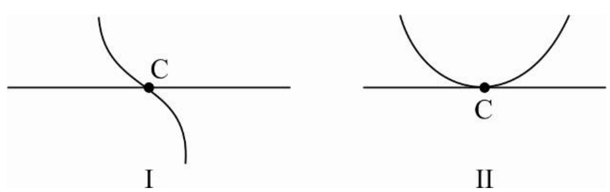
**22.(3)** Given function is defined from $R\to R$ and it is given that function is differentiable at $x=c$ and $f(c)=0$

This is possible in two ways

Here it is obvious that

If $g(x)=|f(x)|$

then $g(x)$ will be differentiable at $x=c$ if second is the case.




**23.(2)** Total number of students are 20

So, $(x+1)^2+(2x-5)+(x^2-3x)+x=20$

$x^2+2x+1+2x-5+x^2-3x+x=20$

$2x^2+2x-24=0$

$x^2+x-12=0$

$x = -4, \ x = 3$

But $x$ is positive so, $x = 3$

Now average marks (mean of the marks) $= \dfrac{16\times 2 + 1\times 3 + 0\times 5 + 7\times 3}{20} = \dfrac{56}{20} = 2.8$

**24.(4)** $\displaystyle\int \frac{dx}{(x^2 - 2x + 10)^2}$

Let $x - 1 = t, \ dx = dt$

$$\int \frac{dt}{(t^2+9)^2}$$

Now $\displaystyle\int 1\cdot \frac{dt}{t^2+9} = \frac{1}{3}\tan^{-1}\frac{t}{3} + c$ ........ (1)

Applying integration by parts on L.H.S of (1)

$$\frac{t}{t^2+9} + 2\int \frac{t^2}{(t^2+9)^2}dt = \frac{1}{3}\tan^{-1}\frac{t}{3} + c$$

$$\frac{t}{t^2+9} + 2\int \frac{dt}{t^2+9} - 18\int \frac{dt}{(t^2+9)^2} = \frac{1}{3}\tan^{-1}\frac{t}{3} + c \quad \Rightarrow \quad \int \frac{dt}{(t^2+9)^2} = \frac{1}{54}\tan^{-1}\frac{t}{3} + \frac{3t}{54(t^2+9)} + c$$

Now put $t = x - 1$

$$\int \frac{dx}{(x^2-2x+10)^2} = \frac{1}{54}\left(\tan^{-1}\frac{(x-1)}{3} + \frac{3(x-1)}{x^2-2x+10}\right) + c$$

Hence $A = \dfrac{1}{54}$ and $f(x) = 3(x-1)$

**25.(3)** $M$ be the mid point of $AC$

So, $M \equiv (2, 1, 0)$

According to the information given in the question

$G$ be the centroid of the triangle.

$G = (2, 4, 2)$

Now $\bar{O}A = 3\hat{i} - \hat{k}$

$\bar{O}G' = 2\hat{i} + 4\hat{i} + 2\hat{k}$

$$\cos(\angle GOA) = \frac{6-2}{\sqrt{10}\sqrt{4+16+4}}$$

$$\cos(\angle GOA) = \frac{4}{\sqrt{240}} = \frac{1}{\sqrt{15}}$$

**26.(4)** According to the given problem

$f(s) = S^2$ and since $S \in [0, 4]$

So, $f(s) \in [0, 16]$

Now we find $f(g(s))$ and $g(f(s))$ and then check the options.

According to the given informations

$0 \le f(g(s)) \le 4$ and $g(f(s)) \in [-4, 4]$

Option 1, 2 and 3 are correct, but option 4 is incorrected.

**27.(2)** It is given that

$$2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$$

$$2^{\sqrt{\sin^2 x - 2\sin x + 5}} \le 4^{\sin^2 y}$$

$$2^{\sqrt{(\sin x - 1)^2 + 4}} \le 4^{\sin^2 y}$$

L.H.S of the inequality will always be greater than or equals to 4 but R.H.S of the inequality will always be less than or equals to 4. So inequation will be true only one case

When $\sin x - 1 = 0$ .......... (i) and $\sin^2 y = 1$ ........ (ii)

So, all the pair $(x, y)$ satisfying (i) and (ii) will also satisfy

$\sin x = |\sin y|$

**28.(4 )** Here it is given that $Q = (0, -1, -3)$ is the image of the point $P$ $(\alpha, \beta, \gamma)$ say in the plane, $3x - y + 4z = 2$

So, $P$ and $Q$ all image of each other in the plane.

$$\frac{\alpha - 0}{3} = \frac{\beta + 1}{-1} = \frac{\gamma + 3}{4} = -2\frac{\left(3\times 0 - (-1) + 4(-3) - 2\right)}{9 + 1 + 16}$$

$$\frac{\alpha}{3} = \frac{\beta + 1}{-1} = \frac{\gamma + 3}{4} = 1$$

$\alpha = 3,\ \beta = -2,\ \gamma = 1$

Hence the point $P$ is $(3, -2, 1)$

Now the area of $\Delta PQR$

$$\frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{QR}\right|$$

$$\overrightarrow{PQ} = -3\hat{i} + \hat{i} - 4\hat{k}$$

$$\overrightarrow{QR} = 3\hat{i} + \hat{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -3 & 1 & -4 \\ 3 & 0 & 1 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \hat{i} - 9\hat{j} - 3\hat{k}$$

So required area $= \frac{1}{2}\sqrt{91}$

**29.(1)** Since $a_1, a_2, a_3 .......a_n$ are in A.P. and it is given that

$$a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$$

$3(a_1 + a_{16}) = 114 \quad \Rightarrow \quad a_1 + a_{16} = 38$

Now $\quad a_1 + a_6 + a_{11} + a_{16} = 2(a_1 + a_{16}) \quad \Rightarrow \quad 76$

**30.(4)** Each born child is equally likely

So, $P(B) = P(G) = \frac{1}{2}$

So required probability $= \dfrac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_2\left(\frac{1}{2}\right)^4} = \dfrac{1}{11}$